MULTIMEDIA UNIVERSITY

STUDENT ID NO

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2016/2017**

## DEM5028 – ENGINEERING MATHEMATICS 2

3 MARCH 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

---

**INSTRUCTIONS TO STUDENT**

1. This question paper consists of **5** pages (**3** pages with **5** questions and 2 pages of appendix.
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

## QUESTION 1 [20 MARKS]

Using suitable integration method, evaluate the following integrals.

a. $\int \frac{8}{x^3 + x} dx$

[7 marks]

b. $\int_1^{e^2} \frac{(\ln x)^3}{x} dx$

[6 marks]

c. $\int_0^1 2xe^{-x} dx$

[7 marks]

## QUESTION 2 [20 MARKS]

a. Find the volume of the solid generated when the region bounded by $y = x^2$, $x = y^2 - 2$, $x = 0$, $x = -1$ and is rotated about x–axis.

[5 marks]

b. Find the solution of the separable differential equation $\frac{dy}{dx} = -\frac{8x}{y}$ that satisfies the initial condition, $y(0) = 2$.

[7 marks]

c. Find the solution for the linear differential equation $\frac{dy}{dx} = x - 2xy$.

[8 marks]

**Continued...**

## QUESTION 3 [20 MARKS]

a. Find whether $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ converge or diverge. If converge, find the limit.

[2 marks]

b. Test the series for convergence or divergence. (Must state the test used)

i). $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

[3 marks]

ii). $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

[3 marks]

c. Consider the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n(x+3)^n}{4^n}$

i). Find the radius of convergence.

[4 marks]

ii). Find the interval of convergence.

[4 marks]

d. Find the second Maclaurin polynomials $(n=2)$ for $e^{-x}$

[4 marks]

## QUESTION 4 [20 MARKS]

a. Given point $A$ is $(-1,2,4)$ and $B$ is $(2,4,6)$ Find:

i). Position vector of $\vec{B}$

[2 marks]

ii). $2\vec{A}+\vec{B}$

[2 marks]

iii). Evaluate $\vec{A}\times\vec{B}$

[4 marks]

b. Find the equation of the plane that contains the line $x=2+4t, y=1+t, z=1+4t$ and the line $x=2-3t, y=3-5t$ and $z=-t$.

[6 marks]

c. Find the parametric and symmetric equation of line passing through $(4,-2,5)$ and parallel to the vector $\overrightarrow{JK}$ where $J(-8,20,1)$ and $K(0,7,0)$.

[6 marks]

**Continued...**

## QUESTION 5 [20 MARKS]

a. For $f(x,y) = x^2 \sin y + y^2 \cos x$. find :

i) $fxx$

[2 marks]

ii) $fxy$

[2 marks]

b. Suppose that $z = x^3 y - y^3 x, x = uv$ and $y = \frac{u}{v}$. Find $\frac{\partial z}{\partial u}$.

[6 marks]

c. Find the critical points of $f(x,y) = 2x^2 - 2xy + y^3$. Determine whether the critical points are local maximums, local minimums or saddle points.

[10 marks]

**End of Page.**

# APPENDIX I: Formulae

## *Integration of common functions*

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

## *Inverse Trigonometry*

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

## *Pythagorean Identities*

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

## *Integration by parts*

$$\int u dv = uv - \int v du$$

## *Areas Between Curves*

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

## *Volume by Washer*

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

$$V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

## *Volume by Cylindrical Shells*

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

## *Linear Differential Equations:*

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

| | |
|---|---|
| **Divergence Test** | If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges. |
| **p-series** | The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$. |
| **Limit Comparison Test** | Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$<br>If $0 < c < \infty$, then both series converge or both diverge. |
| **Alternating Series Test** | If the alternating series<br>$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$<br>Satisfies : i. $b_{n+1} \leq b_n$ for all $n$<br>ii. $\lim_{n\to\infty} b_n = 0$<br>then the series is convergent |
| **Ratio Test** | Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$<br>a. Series converges absolutely if $L < 1$<br>b. Series diverges if $L > 1$ or $L = \infty$<br>c. No conclusion if $L = 1$ |

***Taylor Polynomials***

$$T_n = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

***Vector***

The length of the vector $\mathbf{a} =< a_1, a_2, a_3 >$ is $|a| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$.

If $\theta$ is the angle between the vector **a** and **b**, then $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$

***Cross Product***

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

***Equation of Line***

Vector equation: $\boldsymbol{r = r_0 + tv}$ Parametric equation: $\boldsymbol{x=x_0+at,\ y=y_0+bt,\ z=z_0+ct}$

***Equation of Plane*** $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

***The Chain Rule***

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

***Second Derivatives Test***

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, $(a, b)$ is a critical point of $f$]. Let

$$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

***Moments and Centers of Mass***

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x,y)dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x,y)dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x,y)dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x,y)dA$$

Where the mass:

$$m = \iint_D \rho(x,y)dA$$

***Triple Integrals:*** $\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$